Accelerated expansion of the Universe filled up with the scalar gravitons

Yu. F. Pirogov
Theory Division, Institute for High Energy Physics, Protvino, RU-142281 Moscow Region, Russia

Abstract

The concept of the scalar graviton as the source of the dark matter and dark energy of the gravitaional origin is applied to study the evolution of the isotropic homogeneous Universe. A realistic self-consistent solution to the modified pure gravity equations, which correctly describes the accelerated expansion of the spatially flat Universe, is found and investigated. It is argued that the scenario with the scalar gravitons filling up the Universe may emulate the LCDM model, reducing thus the true dark matter to an artefact.

1 Introduction

According to the present-day cosmological paradigm our Universe is fairly isotropic, homogeneous, spatially flat and experiences presently the accelerated expansion. The conventional description of the latter phenomenon is given by the model with the Λ-term and the cold dark matter (CDM).[1] Nevertheless, such a description may be just a phenomenological reflection of a more fundamental mechanism. A realistic candidate on such a role is presented in the given paper.

In a preceding paper [2], we proposed a modification of the General Relativity (GR), with the massive scalar graviton in addition to the massless tensor one.[2] The scalar graviton was put forward as a source of the dark matter (DM) and the dark energy (DE) of the gravitational origin. In ref. [4], this concept was applied to study the evolution of the isotropic homogeneous Universe. The evolution equations were derived and the plausible arguments in favour of the reality of the evolution scenario with the scalar gravitons were presented.

In the present paper, we expose an explicit solution to the evolution equations in the vacuum, which gives the correct description of the accelerated expansion of the spatially flat Universe. It is shown that the emulation of the LCDM model can indeed be reached as it was anticipated earlier [4]. In Section 2, we first briefly remind the evolution equations in the vacuum filled up only with the scalar gravitons. Then the master equation for the Hubble parameter is presented. Finally, a self-consistent solution of the latter equation, possessing the desired properties, is found and investigated. In the Conclusion, the proposed solution to the DM and DE problems is recapitulated.

[1] Hereof, the LCDM model. For a review on cosmology, see, e.g., ref. [1].
[2] For a brief exposition of such a modified GR, see ref. [3].

2 Accelerated expansion

Evolution equations We consider the isotropic homogeneous Universe without the true DM. Besides, we neglect by the luminous matter missing thus the initial period of the Universe evolution. Then, the vacuum evolution equations look like[3]

$$\begin{aligned} 3\Big(\Big(\frac{\dot a}{a}\Big)^2+\frac{\kappa^2}{a^2}\Big) &= \frac{1}{m_{\rm P}^2}(\rho_{\rm s}+\rho_\Lambda),\\ 2\,\frac{\ddot a}{a}+\Big(\frac{\dot a}{a}\Big)^2+\frac{\kappa^2}{a^2} &= -\frac{1}{m_{\rm P}^2}(p_{\rm s}+p_\Lambda), \end{aligned} \tag{1}$$

with $a(t)$ being the dynamical scale factor of the Universe, t being the comoving time and $\dot a = da/dt$, etc. In the above, κ^2 is proportional to the spatial curvature, with $\kappa^2 = 0$ for the spatially flat Universe. The parameter $m_{\rm P}$ is the Planck mass.

On the r.h.s. of eq. (1), ρ_Λ and p_Λ are the energy density and the pressure corresponding to the cosmological constant Λ: $\rho_\Lambda = -p_\Lambda = m_{\rm P}^2\Lambda \geq 0$. Likewise, $\rho_{\rm s}$ and $p_{\rm s}$ are, respectively, the energy density and pressure of the scalar gravitons:

$$\begin{aligned} \rho_{\rm s} &= f_{\rm s}^2\Big(\frac{1}{2}\dot\sigma^2+3\frac{\dot a}{a}\dot\sigma+\ddot\sigma\Big)+m_{\rm P}^2\Lambda_{\rm s}(\sigma),\\ p_{\rm s} &= f_{\rm s}^2\Big(\frac{1}{2}\dot\sigma^2-3\frac{\dot a}{a}\dot\sigma-\ddot\sigma\Big)-m_{\rm P}^2\Lambda_{\rm s}(\sigma). \end{aligned} \tag{2}$$

Here, $f_{\rm s} = \mathcal{O}(m_{\rm P})$ is a constant with the dimension of mass entering the kinetic term of the scalar graviton field σ. The latter in the given context looks like

$$\sigma = 3\ln\frac{a}{\tilde a}, \tag{3}$$

with $\tilde a(t)$ being a nondynamical scale factor given a priori. The σ-field is defined up to an additive constant. Without any loose of generality, we can fix the constant by the asymptotic condition: $\sigma(t)\to 0$ at $t\to\infty$.

In eq. (2), we put

$$V_{\rm s}+\partial V_{\rm s}/\partial\sigma \equiv m_{\rm P}^2\Big(\Lambda_{\rm s}(\sigma)+\Lambda\Big), \tag{4}$$

where $V_{\rm s}(\sigma)$ is the scalar graviton potential. More particularly, we put

$$V_{\rm s} = V_0+\frac{1}{2}m_{\rm s}^2 f_{\rm s}^2(\sigma-\sigma_0)^2+\mathcal{O}\Big((\sigma-\sigma_0)^3\Big), \tag{5}$$

with σ_0 being a constant, $f_{\rm s}(\sigma-\sigma_0)$ the physical field of the scalar graviton and $m_{\rm s}$ the mass of the latter. By their nature, $\Lambda_{\rm s}$ and Λ are quite similar. To make the division onto these two parts unambiguous we normalize $\Lambda_{\rm s}$ by an additive constant so that $\Lambda_{\rm s}(0) = 0$. Clearly, we get from eq. (2) that $\rho_{\rm s}+p_{\rm s} = f_{\rm s}^2\dot\sigma^2$. Here, the contribution of $\Lambda_{\rm s}$ exactly cancels what is quite similar to the relation $\rho_\Lambda+p_\Lambda = 0$. So, the contribution of $\Lambda_{\rm s}$ is a kind of the dark energy. In what follows, we put $\sigma_0 = 0$ and $V_0 = m_{\rm P}^2\Lambda$, with $\sigma\to 0$ at $t\to\infty$ becoming the ground state.

The nondynamical functions $V_{\rm s}$ and $\tilde a$ being the two characteristics of the vacuum are not quite independent. More particularly, adopting the isotropic homogeneous ansatz for the solution of the gravity equations, with only one dynamical variable a, we tacitly put a consistency relation between $\tilde a$ and $V_{\rm s}$. As a result, only one combination of the two lines of eq. (1) is the true equation of evolution, with the second independent combination giving just the required consistency condition.

[3]We refer the reader to ref. [4] for more details.

Master equation In what follows, we restrict ourselves by the case of the spatially flat Universe, $\kappa = 0$. Subtracting the first line of eq. (1) from the second one and accounting for eq. (2) we get the relation

$$\dot{H} = -\frac{1}{4}\alpha\dot{\sigma}^2, \tag{6}$$

where $H \equiv \dot{a}/a$ is the Hubble parameter and

$$\alpha = 2\Big(\frac{f_{\rm s}}{m_{\rm P}}\Big)^2. \tag{7}$$

We assume that $\alpha = \mathcal{O}(1)$. Substituting $\dot{\sigma}$ given by eq. (6) into the first line of eq. (1) we get the integro-differential master equation for the Hubble parameter:

$$H^2 = -\frac{1}{3}\Big(\frac{\sqrt{\alpha}}{2}\frac{\ddot{H}+6H\dot{H}}{\sqrt{-\dot{H}}} + \dot{H}\Big) + \frac{1}{3}\Big(\Lambda_{\rm s}(\sigma) + \Lambda\Big). \tag{8}$$

where it is to be understood

$$\sigma = \frac{2}{\sqrt{\alpha}}\int_\infty^t \sqrt{-\dot{H}(\tau)}\,d\tau. \tag{9}$$

Remind that we assume $\sigma(t) \to 0$ at $t \to \infty$. Equations (8) and (6) supersede the pair of the original evolution equations (1).

Self-consistent solution Let us put in what follows $\Lambda_{\rm s} \equiv 0$. This will be justified afterwards. Iterating eq. (8), with Λ considered as a perturbation, we can get the solution with any desired accuracy. In particular, substituting into the r.h.s. of eq. (8) the solution $H = \alpha/t$ from the zeroth approximation ($\Lambda = 0$) we get the first approximation as follows

$$H^2 = \Big(\frac{\alpha}{t}\Big)^2 + \frac{\Lambda}{3} + \begin{cases} \mathcal{O}(1) & \text{at } t \to 0 \;, \\ \mathcal{O}(1/t^3) & \text{at t} \to \infty, \end{cases} \tag{10}$$

or otherwise

$$H^2 = \frac{\alpha^2}{t_\Lambda^2}\Big((t_\Lambda/t)^2 + 1\Big) \simeq \begin{cases} \alpha^2/t^2 & \text{at } t/t_\Lambda < 1, \\ \alpha^2/t_\Lambda^2 & \text{at } t/t_\Lambda > 1, \end{cases} \tag{11}$$

with

$$t_\Lambda = \frac{\alpha}{\sqrt{\Lambda/3}} \tag{12}$$

being the characteristic time of the evolution of the Universe. Numerically, $t_\Lambda \sim 10^{10}$yr is of order the age of the Universe. Equation (11) is the basis for the qualitative discussion in what follows.

Integrating eq. (11) we get the scale factor as follows:

$$\ln\frac{a}{a_0} = \alpha\Big[\frac{t}{t_\Lambda}\sqrt{\Big(\frac{t_\Lambda}{t}\Big)^2 + 1} - \ln\Big(\frac{t_\Lambda}{t} + \sqrt{\Big(\frac{t_\Lambda}{t}\Big)^2 + 1}\Big)\Big] \sim \begin{cases} \alpha\ln(t/t_\Lambda) & \text{at } t/t_\Lambda < 1, \\ \alpha t/t_\Lambda & \text{at } t/t_\Lambda > 1, \end{cases} \tag{13}$$

where a_0 is an integration constant.[4] Explicitly, the scale factor looks like

$$a \sim \begin{cases} (t/t_\Lambda)^\alpha & \text{at } t/t_\Lambda < 1, \\ \exp(\alpha t/t_\Lambda) & \text{at } t/t_\Lambda > 1, \end{cases} \tag{14}$$

[4] To phenomenologically account for the effect of the initial inflation we could formally shift the origin of time: $t \to t + t_0$, with $t_0 > 0$.

with t_Λ bordering thus the epoch of the the power law expansion from the epoch of the exponential expansion. Equation (13) gives the two-parametric representation for the scale factor of the acceleratedly expanding Universe after the initial period.

With account for eq. (9) the σ-field behaves as

$$\sigma = -\int_{(t/t_\Lambda)^2}^{\infty} \frac{d\xi}{\xi(1+\xi)^{1/4}} \sim \begin{cases} 2\ln(t/t_\Lambda) & \text{at } t/t_\Lambda < 1, \\ -4\sqrt{t_\Lambda/t} & \text{at } t/t_\Lambda > 1. \end{cases} \tag{15}$$

Note that at $t_\Lambda \to \infty$ or, equivalently, $\Lambda \to 0$ the integral above diverges and the σ-field can not be normalized properly. $\Lambda \neq 0$ is thus necessary as a regulator in the theory. Now, the consistency condition looks like

$$\tilde{a} = a \exp\left(-\sigma/3\right) \sim \begin{cases} (t/t_\Lambda)^{\alpha-2/3} & \text{at } t/t_\Lambda < 1, \\ \exp\left(\alpha t/t_\Lambda\right) & \text{at } t/t_\Lambda > 1. \end{cases} \tag{16}$$

Clearly, Λ should be already presupposed in $\tilde{a}$. Note that in the case $\alpha = 2/3$, the parameter $\tilde{a}$ is approximately constant at $t/t_\Lambda < 1$.

Substituting equations (11) and (6) into the first line of eq. (2) we can explicitly verify that

$$\frac{1}{m_{\mathrm{P}}^2}\rho_{\mathrm{s}} = \frac{3\alpha^2}{t^2}. \tag{17}$$

This is to be anticipated already from the relation $\rho_\Lambda/m_{\mathrm{P}}^2 = \Lambda$, as well as eq. (10) and the first line of eq. (1). Clearly, ρ_{s} is positive. At $t/t_\Lambda < 1$ we get from equations (14) and (17) that $\rho_{\mathrm{s}} a^3 \sim t^{3\alpha-2}$. In the case $\alpha = 2/3$, we have $\rho_{\mathrm{s}} \sim 1/a^3$ as it should be for the true CDM. On the other hand, the pressure of the scalar gravitons is as follows

$$\frac{1}{m_{\mathrm{P}}^2} p_{\mathrm{s}} = -\frac{3\alpha^2}{t^2} + \frac{2\alpha}{t_\Lambda^2}\frac{(t_\Lambda/t)^3}{\sqrt{(t_\Lambda/t)^2+1}} \simeq \begin{cases} 3\alpha(2/3-\alpha)/t^2 - \alpha/t_\Lambda^2 & \text{at } t/t_\Lambda < 1, \\ (2\alpha/t_\Lambda^2)(t_\Lambda/t)^3 & \text{at } t/t_\Lambda > 1. \end{cases} \tag{18}$$

At the same conditions as before, the pressure is $p_{\mathrm{s}}/m_{\mathrm{P}}^2 = -\Lambda/2$, being near constant though not zero as it should be anticipated for the true CDM. Nevertheless, we see that the value $\alpha = 2/3$ is exceptional in many respects. Conceivably, such a value is distinguished by a more fundamental theory.

Introducing the critical energy density $\rho_{\mathrm{c}} = 3m_{\mathrm{P}}^2 H^2$, we get for the partial energy densities $\Omega_{\mathrm{s}} = \rho_{\mathrm{s}}/\rho_{\mathrm{c}}$ and $\Omega_\Lambda = \rho_\Lambda/\rho_{\mathrm{c}}$, respectively, of the scalar gravitons and the Λ-term the following:

$$\Omega_{\mathrm{s}} = \frac{1}{1+(t/t_\Lambda)^2} \simeq \begin{cases} 1-(t/t_\Lambda)^2 & \text{at } t/t_\Lambda < 1, \\ (t_\Lambda/t)^2 & \text{at } t/t_\Lambda > 1, \end{cases} \tag{19}$$

with $\Omega_\Lambda = 1 - \Omega_{\mathrm{s}}$. Note that $\Omega_{\mathrm{s}} = \Omega_\Lambda = 1/2$ at $t/t_\Lambda = 1$. Presently, we have $\Omega_{\mathrm{s}}/\Omega_\Lambda \simeq 1/3$ and thus the respective time t, in the neglect by the effect of the initial inflation, is somewhat larger t_Λ.

Finally, the condition $\Lambda_{\mathrm{s}} = 0$ adopted earlier can be justified as follows. First of all, Λ_{s} is indeed negligible at $t \to \infty$ due to $\sigma \to 0$ and $\Lambda_{\mathrm{s}}(0) = 0$. On the other hand, at $t \sim t_\Lambda$ we have $|\sigma| \sim 1$ and hence $\Lambda_{\mathrm{s}} \sim m_{\mathrm{s}}^2$. For Λ_{s} to be negligible in this region, too, we should require $m_{\mathrm{s}} \leq \sqrt{\Lambda} \sim 1/t_\Lambda$. Nevertheless, in the early period of evolution when $|\sigma| > 1$ the contribution of Λ_{s} may be significant. The parameter $\alpha = 2/3$ being fixed the theory may be terminated just by two mass parameters: the ultraviolet m_{P} and the infrared $t_\Lambda^{-1} \sim \sqrt{\Lambda}$ or, otherwise, m_{s}.

3 Conclusion

To conclude, let us recapitulate the proposed solution to the DM and DE problems in the context of the evolution of the Universe. According to the viewpoint adopted, there is neither true DM nor DE in the Universe (at least, in a sizable amount). Instead, the field σ of the scalar graviton serves as a common source of both the DM and DE of the gravitational origin. DM is represented by the derivative contribution of σ, with DE being reflected by the derivativeless contribution. In this, the constant part of the latter contribution corresponds to the conventional Λ-term, while the σ-dependent part corresponds to DE. The latter is less important than the Λ-term at present, becoming conceivably more crucial at the early time.

The self-consistent evolution of the Universe may be considered as the transition of the "sea" of the scalar gravitons, produced in the early period, from the excited state with $|\sigma| > 1$ to the ground state with $\sigma = 0$. The ground state is characterized by the cosmological constant Λ which, in turn, predetermines the characteristic evolution time of the Universe, $t_\Lambda \sim 1/\sqrt{\Lambda}$. The scenario is in the possession to naturally describe the accelerated expansion of the spatially flat Universe, correctly emulating thus the conventional LCDM model. The more complete study of the scenario, the initial period of the evolution including, is in order.

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References

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